## Hemi-spherical projections

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## Introduction

I have been so confused for a number of years regarding the various types of hemispherical projections and how to get from one to the other. This document attempts to sum up and put in writing what I presently regard as being The Truth.

This document covers both the situation where you wish to understand the projection when taking images of a reflective sphere and of a diffuse sphere, as well as when you want to understand the projection behind images obtained from a raytracer such as RADIANCE, or images acquired with a fish eye lens.

## The geometry of imaging diffuse and reflective spheres

This section deals with the geometry behind images taken of diffuse or reflective spheres with a normal lens. It is assumed that the distance between the camera and the sphere is large enough to assume that the projection to the image plane is orthogonal, i.e. the rays are parallel.

## Diffuse sphere

Figure 1 close shows an image of a diffuse sphere before and after cropping.


Figure 1. Left: A synthesized to down image of a small setup with a diffuse sphere in the middle. We wish the relate the pixels on the diffuse sphere to directions in 3D space. Right: the image cropped to only contain the diffuse sphere. The speckles are due to Monte Carlo noise in the rendering (could be alleviated with subsampling a high resolution rendering.

Under the assumption that the image of the diffuse sphere is approximately orthogonal all rays from the camera to points on the sphere surface are parallel. Figure 2 shows the geometry of such a situation. A ray intersects the sphere at a certain point, and the sphere, at this point, has a normal vector, $\mathbf{N}$. The azimuth, $\phi$, of this normal is easily found from the direction in the image from the sphere center to the intersection point/pixel. If the pixel coordinates of the intersection point are $u$ and $v$ (relative to a coordinate system with origin in the cropped image center, with $u$-axis pointing right, and $v$-axis pointing up), and if the radius of the cropped sphere is dmax pixels:

$$
\varphi=\tan ^{-1} \frac{\mathrm{v}}{\mathrm{u}}
$$

The inclination angle, $\theta$, from Zenith, i.e. vertically up straight into camera, is found as illustrated in Figure 2 :

$$
\theta=\sin ^{-1} \frac{d}{d_{\max }}
$$



Figure 2. The geometry of an orthogonal projection of a diffuse sphere.

## Reflective sphere

The reflective sphere case is very similar to the diffuse sphere case, except from one crucial difference. In the reflective case the inclination angle grows twice as fast with the radial distance to the intersection point. This is illustrated in Figure 3, where the inclination of the surface normal is termed $\theta$ as in the diffuse case, but the inclination angle of the reflection direction, $\omega$, is twice that:


Figure 3. The inclination angle grows at double rate due to the reflection direction being mirrored around the sphere surface normal.

## RADIANCE

The radiance rendering package offers two hemi-spherical projection modes as an option for rpict (rpict -vth or rpict-vta).

## Hemispherical projection (option -vth)

Excerpt from manpages.pdf: A hemispherical fisheye is a projection of the hemisphere onto a circle. The maximum view angle for this type is 180 degrees.

In such a projection (called equi-solid angle) the relationship between the radial distance the image center to a given pixel (d) and the corresponding ray angle (with optical axis) can be expressed as:

$$
\theta=\sin ^{-1} \frac{d}{d_{\max }}
$$



Figure 4. Left: 180 degree hemispherical projection of sky dome and sun (at 45 degrees from vertical, due south). Center of sun disk is at $d=226$, and $d \max =320$, yielding and angle from vertical of $\arcsin (226 / 320)=44.9$ degrees. Right: spheres at every 10 degrees from horizontal to vertical.

## Angular fish eye projection (option -vta)

Excerpt from manpages.pdf: An angular fisheye view is defined such that distance from the center of the image is proportional to the angle from the central view direction. An angular fisheye can display a full 360 degrees.

In such a projection (called equidistant) the relationship between the radial distance to the image center to a given pixel (d) and the corresponding ray angle (with optical axis) can be expressed as (where FOV is the Field-Of-View in radians as expressed in degrees in the .vp file):

$$
\theta=F O V \cdot \frac{d}{d_{\max }}
$$



Figure 5. Left: 180 degree angular fish eye projection of sky dome and sun (at 45 degrees from vertical, due south). Center of sun disk is at half the distance between image center and circle circumference. Right: spheres at every 10 degrees from horizontal to vertical. 8 spheres are visible.

## Sigma 180 degree FOV fish eye lens

According to searches on the internet, the 180 degree FOV lens for the Canon 1Ds Mark II camera provides an equi-solid angle projection, where the mapping function is described as:

$$
\theta=\sin ^{-1} \frac{d}{d_{\max }}
$$

The typical mapping function description found on Wikipedia described inversely as follows:

$$
r=2 \cdot f \cdot \sin \frac{\theta}{2}
$$

We have done experiments that clearly indicate that the projection is equidistant projection of the expression (identical to angular fish eye projection in RADIANCE):


Figure 6: Composite image taken with Sigma 180 degree Field-of-View lens showing a sequence of exposures of a table tennis ball, acquired by rotating the camera by 5 degrees around optical center between each exposure. The table tennis ball projections are equidistant clearly demonstrating the type of projection to be equidistant. There are 37 visible table tennis ball impressions in this composite.

## Remapping equi-distant images to lat-long

This section is based on HDRShop, version 3.0
Equi-distant projections, such Sigma Fish eye lens images and angular fish eye images (option -vta) from RADIANCE, must be converted in HDRShop by setting source format to Mirrored Ball Closeup and setting target format to lat-long.


Figure 7. Angular fish eye lens image (equi-distant projection) from RADIANCE remapped to lat-long using Mirrored Ball Closeup source setting. Notice that spheres maintain their equi-distance. Fish eye images from Sigma 180 degree FOV lens must be treated in the same manner. The image has been shown with border to stress the fact the the top part of the remapping is the sphere corresponding to straight up. 8 spheres are visible.

## Remapping equi-solid angle images to lat-long HDRShop version 3.0

Equi-solid angle projections, such as hemispherical projection images (option -vth) from RADIANCE, must be converted in HDRShop by setting source format to Diffuse Ball and setting target format to lat-long.


Figure 8. Hemispherical projection (equi-solid angle) image from RADIANCE remapped to lat-long using Diffuse Ball source setting. Spheres are equi-distant. The image has been shown with border to stress the fact the top part of the remapping is the sphere corresponding to straight up. 8 spheres are visible.

## Remapping image of diffuse sphere to lat-long

HDRShop version 3.0
As a special case of all of the above we are sometimes interested in re-mapping a cropped image of a diffuse sphere to lat-long format. Until HDRShop 3.0 this has not been possible.


Figure 9. Top: simulated approximately orthogonal projection of a diffuse sphere (for irradiance measurements). Bottom left: remapped to lat-long using Diffuse Ball source setting (correct). Bottom right: remapped to lat-long using Mirrored Ball Closeup source setting (wrong).

